

Part I – Linear Equations

Before we really get started with this section, two definitions are necessary.

Linear equations are equations that represent lines when they are graphed in the Cartesian plane.

The standard form of a linear equation has the form $Ax + By = C$, where A , B , and C are integral constants (or numbers that are integers and will not change once you have started your problem), and A is not negative.

Linear equations are the nicest-looking kind of equation: They do not have variables raised to any power other than 1, they do not have any variables in the denominator of fractions or under square root symbols, and the x 's are not multiplied by the y 's. Now, let's look at some examples to illustrate these two definitions.

Example 1: Which of the following are linear equations? (a) $x^2 - 2y = 4$, (b) $x^5 = 4 + y$, (c) $6x = -2y - 1.5$, (d) $y = 3$, (e) $x + 5 = 4$, (f) $x + \sqrt{2} = y$, (g) $x = \frac{1}{2}y$

Only (a) and (b) are not linear equations because equations (c) through (g) meet the criteria for linear equations discussed above. Furthermore, as you will see in Example 3, all of these equations produce straight lines when they are graphed.

Example 2: Write the linear equation $2y = -x + 1$ in standard form.

When we add x to both sides, we find that this equation can be rearranged into the form $x + 2y = 1$, which is the standard form of this equation. (Note that we could refer back to the definition of the standard form of a linear equation and say that $A = 1$, $B = 2$, and $C = 1$. Therefore, A , B , and C are integers, and A is not negative.)

Example 3: Write the linear equation $y = \frac{2}{3}x - \frac{5}{4}$ in standard form.

To solve this problem, we will start by getting the variables on one side and the number on the other side. To do this, we will subtract $\frac{2}{3}x$ from both sides. This

gives us $-\frac{2}{3}x + y = -\frac{5}{4}$. Next, since the number in front of the x cannot be

negative, we will multiply both sides by -1 , which gives us the equation

$\frac{2}{3}x - y = \frac{5}{4}$. Now, since all the constants must be integers, we will multiply both

sides by the least common denominator of the fractions, which, in this case, is 12.

This gives us the equation $8x - 12y = 15$. Since this is an equation in standard form, the equation $8x - 12y = 15$ is our final answer.

Example 4: Graph each of the linear equations from Example 1.

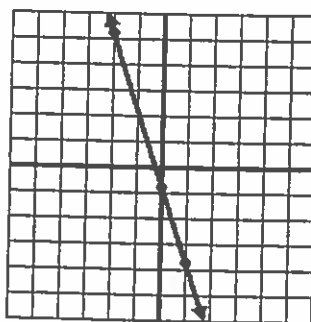
(c) To graph $6x = -2y - 1.5$, we could start by making a chart like we did in the last chapter. For this example, we will pick x to be -2 , 0 , and 1 . Therefore, our chart starts out looking like this:

x	y
-2	
0	
1	

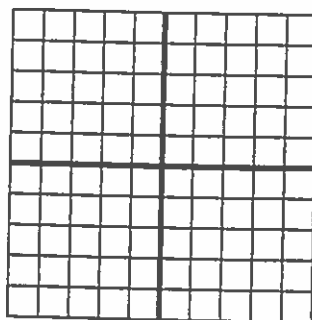
Now, we can plug each of these x -values in and find their corresponding y -values: $6(-2) = -2y - 1.5$ tells us that $y = 5.25$, $6(0) = -2y - 1.5$ tells us that $y = -0.75$, and $6(1) = -2y - 1.5$ tells us that $y = -3.75$. Therefore, we our finished chart looks like this:

x	y
-2	5.25
0	-0.75
1	-3.75

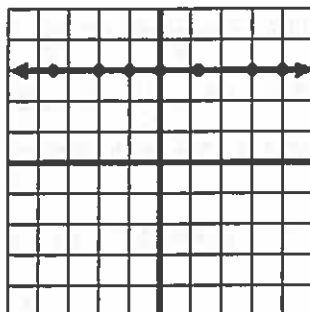
Finally, we can plot these points and draw a line through them:



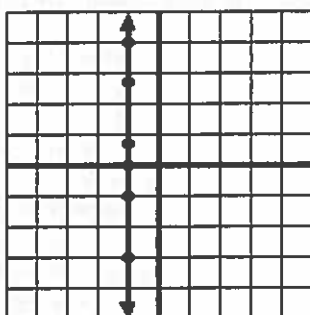
(d) The graph of the line $y = 3$ is very different from anything we have discussed up to this point. In the graph below, plot at least six points where you know that the y -value is 3.



Do you see the straight line? Your final graph should look something like the following:



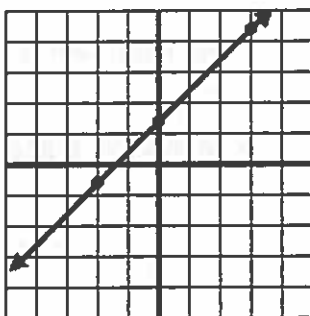
(e) The graph of $x + 5 = 4$ is somewhat similar to the graph of the line from (d). First of all, note that an equivalent form of this equation is $x = -1$. Therefore, we must find several points where we know that the x -value is equal to -1 . This tells us that the graph of $x + 5 = 4$ looks like the graph below.



(f) To graph the line represented by the equation $x + \sqrt{2} = y$, we will start by making our chart again. This time, we will let x equal -2 , 0 , and 3 . You can use a calculator to tell you that $\sqrt{2}$ is approximately equal to 1.41 . This tells us that we can fill in our chart like this:

x	y
-2	≈ -0.59
0	≈ 1.41
3	≈ 4.41

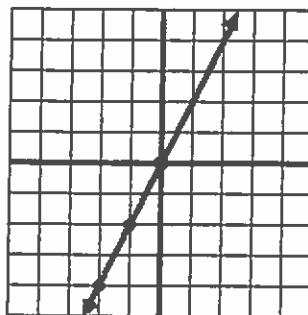
This tells us that our graph looks like this: .



(g) To graph the line represented by the equation $x = \frac{1}{2}y$, we will start by making our chart again. This time, we will pick x to be -2 , -1 , and 0 . Now, we can plug in these x -values and find the following: $-2 = \frac{1}{2}y$ tells us that $y = -4$, $-1 = \frac{1}{2}y$ tells us that $y = -2$, and $0 = \frac{1}{2}y$ tells us that $y = 0$. This gives us the following chart:

x	y
-2	-4
-1	-2
0	0

Finally, we can see that the graph of the equation $x = \frac{1}{2}y$ looks like this:



Problems:

- Does the equation $2y + 4x = 1$ represent a line? _____
- Does the equation $8x^2 - y = 3$ represent a line? _____
- Does the equation $4y^3 - x = 1$ represent a line? _____
- Does the equation $y = 5$ represent a line? _____
- Does the equation $y^2 = x - 5$ represent a line? _____
- Does the equation $\frac{x}{4} = 5$ represent a line? _____
- Does the equation $\frac{y}{8} = -2$ represent a line? _____
- Does the equation $3y - \frac{5}{x} = 0$ represent a line? _____
- Does the equation $\frac{y}{2} = \frac{x}{5} + 1$ represent a line? _____
- Does the equation $-2x + 3y = 5$ represent a line? _____

11. Does the equation $x = -104$ represent a line? _____

12. Does the equation $xy = -6$ represent a line? _____

13. Does the equation $\frac{4}{x} + 2 = y$ represent a line? _____

14. Does the equation $\frac{x}{5} + \frac{y}{-1} = -3$ represent a line? _____

15. Does the equation $4xy = 8$ represent a line? _____

16. Write the linear equation $y = 4x + 1$ in standard form. _____

17. Write the linear equation $y + 5 = -x$ in standard form. _____

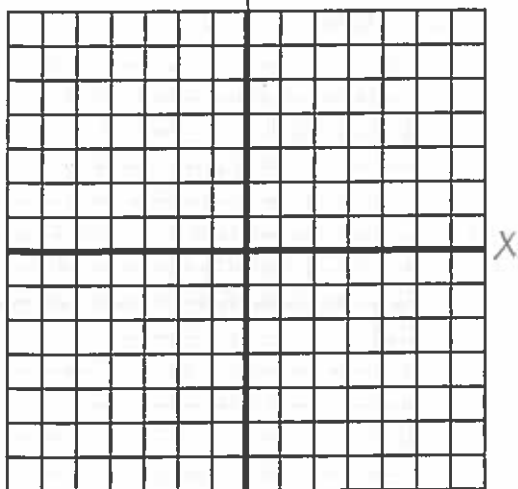
18. Write the linear equation $y = \frac{1}{4}x + 2$ in standard form. _____

19. Write the linear equation $y = -\frac{2}{3}x + 1$ in standard form. _____

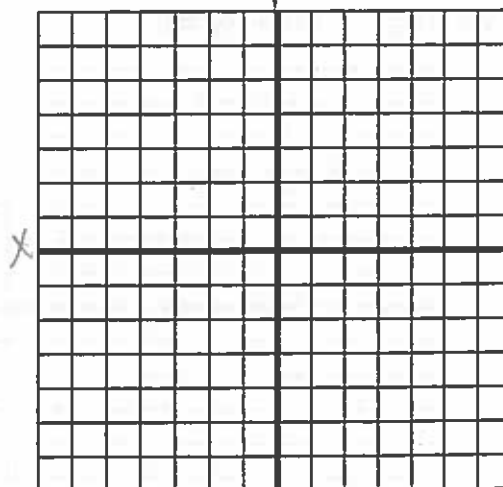
20. Write the linear equation $\frac{1}{5}y = \frac{3}{5}x$ in standard form. _____

21. Write the linear equation $\frac{5}{6}y - \frac{1}{2}x - 4 = 0$ in standard form. _____

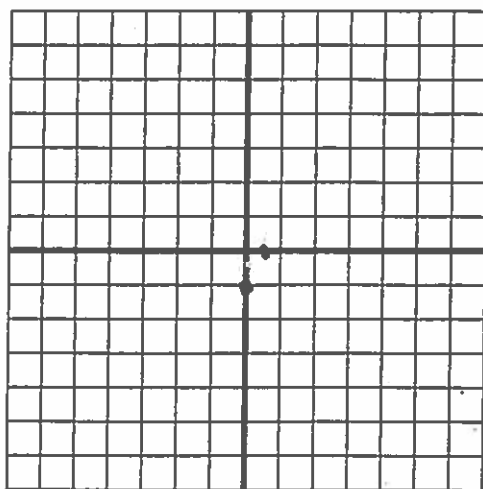
22. Graph: $5x + 3y = 0$



23. Graph: $4y - 2x = 3$

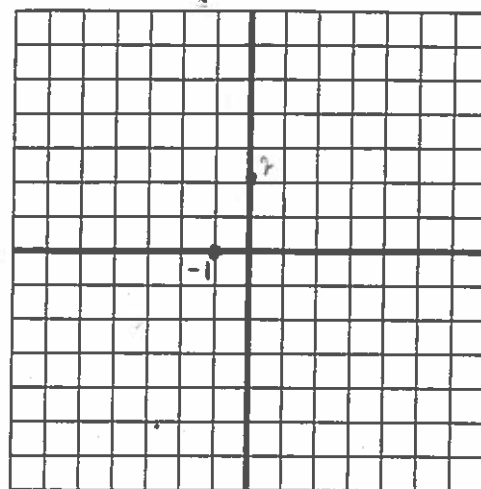


24. Graph: $2x - y = 1$



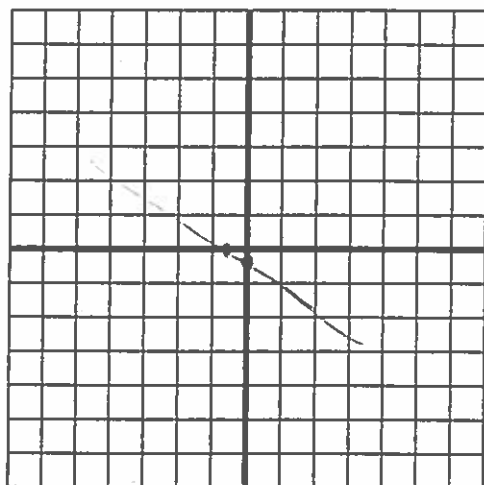
$x(1, 0)$
 $y(0, -1)$

27. Graph: $2x = y - 2$



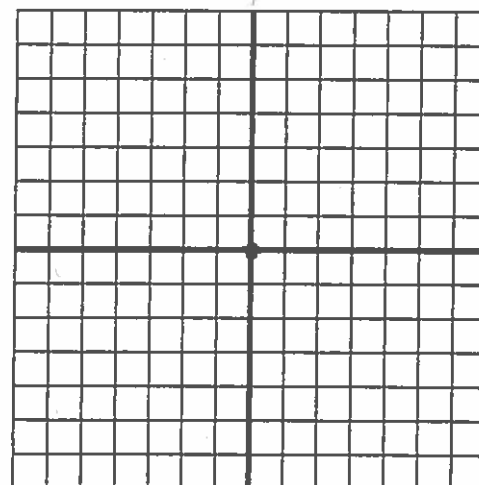
$x(-1, 0)$
 $y(0, 2)$

25. Graph: $2x + 3y = -1$



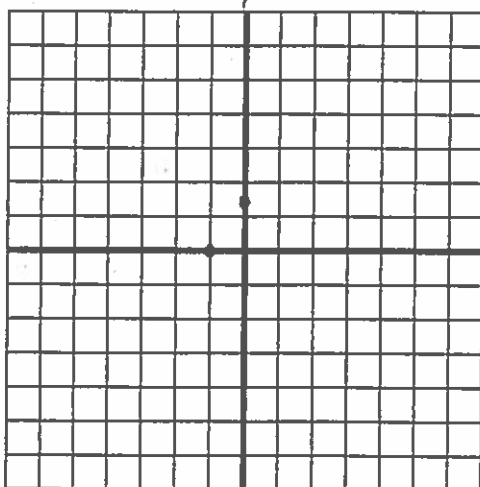
$x(-\frac{1}{2}, 0)$
 $y(0, -\frac{1}{3})$

28. Graph: $0.5x + 0.3y = 0$



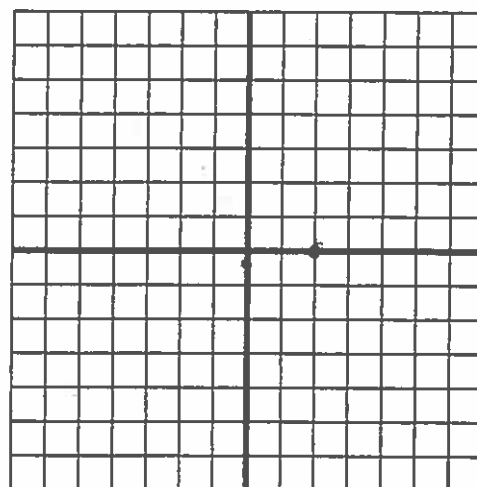
$x(0, 0)$
 $y(0, 0)$

26. Graph: $-3x + 2y = 3$



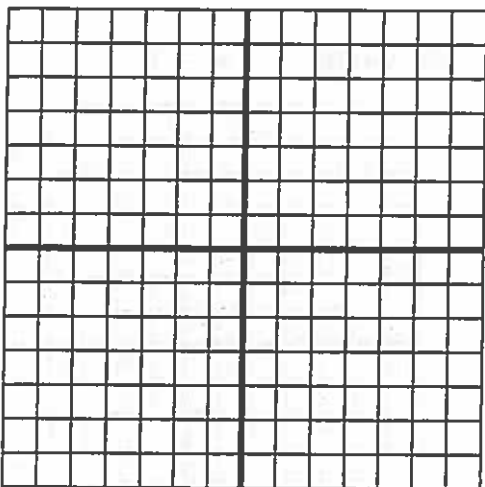
$x(0, 1.5)$
 $y(-1, 0)$

29. Graph: $x - 3y = 2$

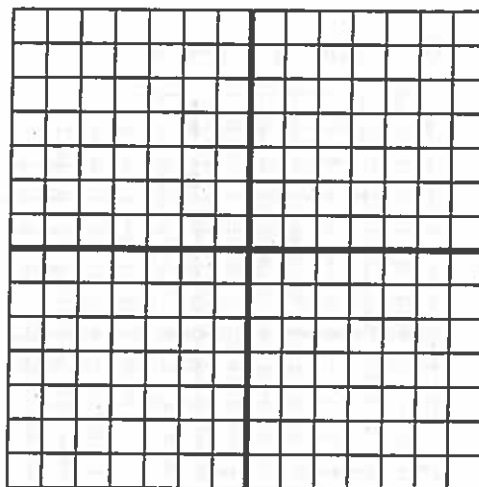


$x(2, 0)$
 $y(0, -\frac{2}{3})$

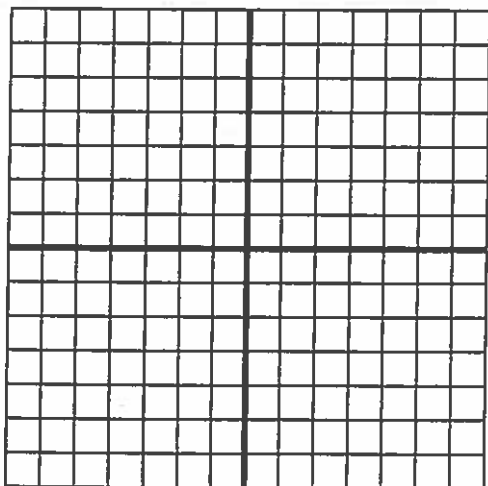
30. Graph: $-x + 2y = 1$



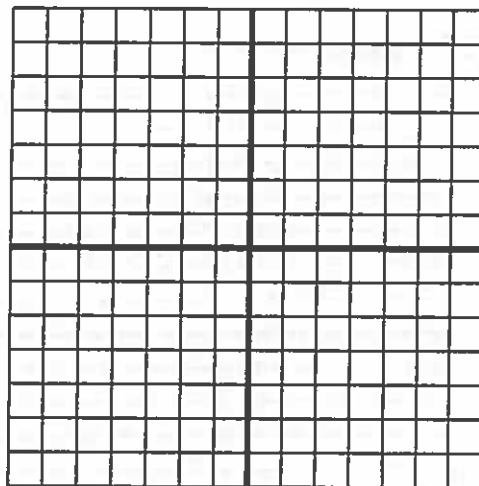
33. Graph: $y = -3$



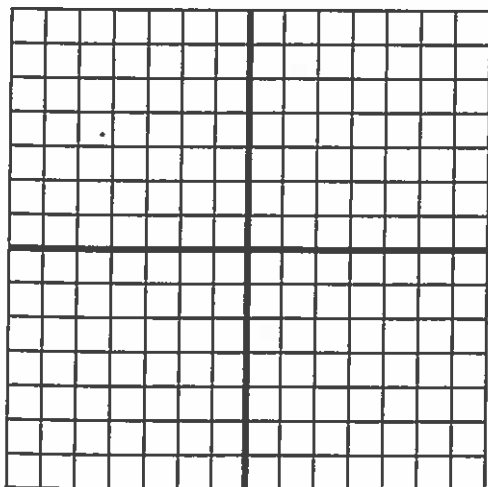
31. Graph: $y = 5$



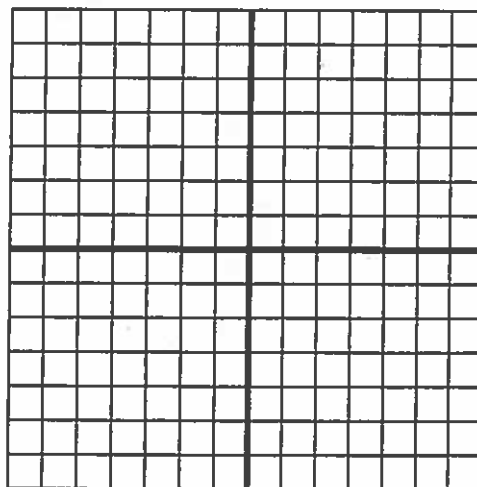
34. Graph: $y = 1$



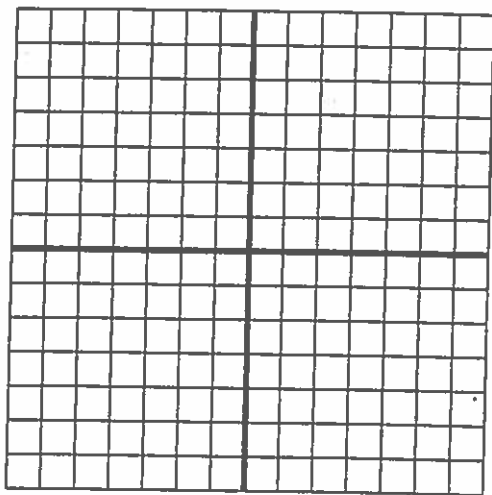
32. Graph: $x = -2$



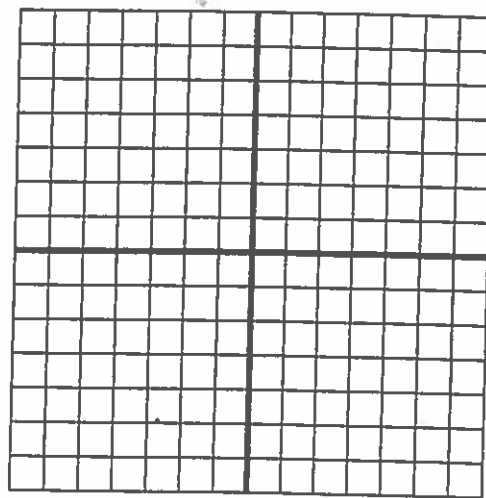
35. Graph: $x + 3 = 1$



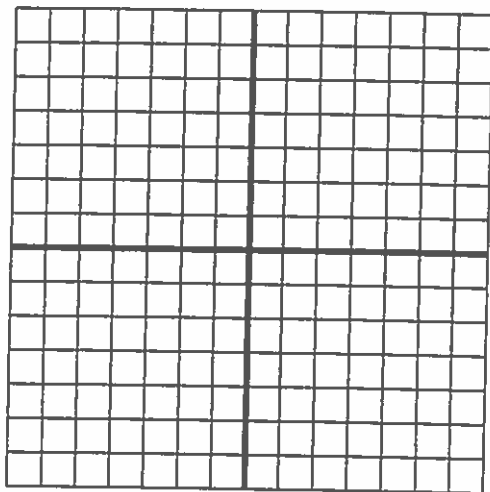
36. Graph: $y - 1 = -4$



38. Graph: $x + 2 = -1$



37. Graph: $x - 7 = -1$

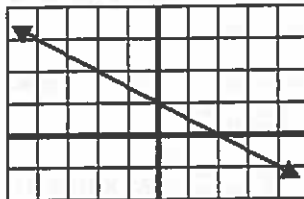


Part II – Introducing Slope and Intercepts

We will start off this section with a definition.

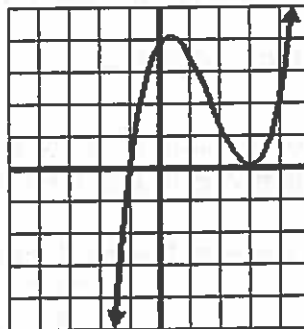
The x-intercepts are the points where a graph intercepts the x-axis (or, equivalently, where $y = 0$). Similarly, the y-intercepts are the points where a graph intercepts the y-axis (or, equivalently, where $x = 0$).

Example 1: In the graph below, what are the x- and y-intercepts?



In this case, the y-intercept is 1 (because the y-value is 1 when the line crosses the y-axis), and the x-intercept is 2 (because the x-value is 2 when the line crosses the x-axis).

Example 2: In the graph below, what are the x- and y-intercepts?

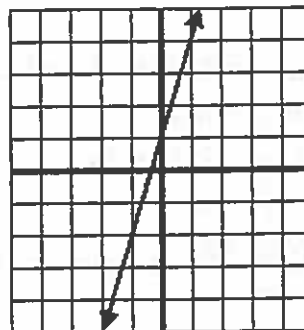


In this case, there are actually 2 x-intercepts: -1 and 3 (note that $y = 0$ both when $x = -1$ and when $x = 3$). The only y-intercept is about 3.75.

Incidentally, the x-intercept(s) are also called the root(s) of the equation.

If the graph of an equation is a line, we can talk about its steepness. The slope of a line is defined by $\frac{\text{rise}}{\text{run}}$, or, equivalently, $\frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are two points on the line. Let's look at some examples.

Example 3: Find the slope of the line below.



For this graph, the slope of this line is 3 (or, in an equivalent fraction form, $\frac{3}{1}$).

To see why, pick two points with integer coordinates on the graph. To get from the first point to the second point, you must do one of two things:

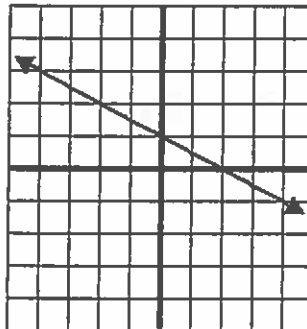
- (1) Rise 3 and run 1, rise 3 and run 1, rise 3 and run 1,... until you get to your second point, or
- (2) Rise -3 and run -1, rise -3 and run -1, rise -3 and run -1,... until you get to your second point

Either way, your $\frac{\text{total rise}}{\text{total run}}$ is going to be a fraction that reduces to $\frac{3}{1}$ (or, equivalently, 3).

You can also find the slope using the formula $\frac{y_2 - y_1}{x_2 - x_1}$. For this example, we'll pick the point (0, 1) to be the (x_1, y_1) and (-1, -2) to be the (x_2, y_2) . This tells us that we can say that the slope equals $\frac{-2-1}{-1-0} = \frac{-3}{-1} = \frac{3}{1}$, or 3.

Either of these two methods will give you the same answer, and picking two different points will also give you the same answer.

Example 4: Find the slope of the line below.



In this graph, the slope is $-\frac{1}{2}$. To see why, pick 2 points with integer coordinates on the graph. To get from the first point to the second point, you must do one of two things:

- (1) Rise -1 and run 2, rise -1 and run 2, rise -1 and run 2,... until you get to your second point, or
- (2) Rise 1 and run -2, rise 1 and run -2, rise 1 and run -2,... until you get to your second point

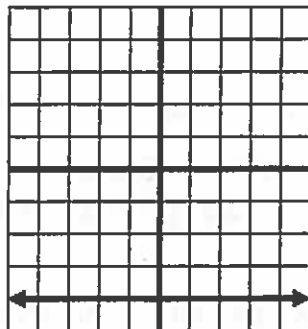
Either way, your $\frac{\text{total rise}}{\text{total run}}$ is going to be a fraction that reduces to $-\frac{1}{2}$.

You can also find the slope using the formula $\frac{y_2 - y_1}{x_2 - x_1}$. For this example, we'll pick the point $(-4, 3)$ to be the (x_1, y_1) and $(2, 0)$ to be the (x_2, y_2) . So we can say that the slope equals $\frac{0 - 3}{2 - (-4)} = \frac{-3}{6} = -\frac{1}{2}$.

Once again, note that either method will give you the same answer, and picking 2 different points will also give you the same answer.

Example 5: Find the slope of the line represented by the equation $y = -4$.

We will start by graphing this line:

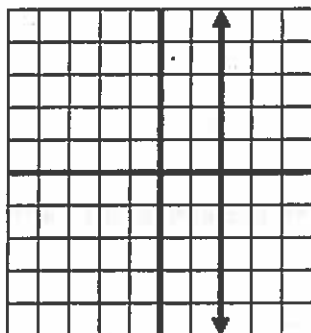


Now, pick any two points on this line. This time, $\frac{\text{total rise}}{\text{total run}}$ is a fraction that reduces to zero.

If you prefer, you can still use the formula slope = $\frac{y_2 - y_1}{x_2 - x_1}$. For this example, we will choose the (x_1, y_1) to be the point $(-2, -4)$, and the (x_2, y_2) will be the point $(3, -4)$. This tells us that the slope of this line is equal to $\frac{-4 - (-4)}{3 - (-2)} = \frac{0}{5}$, which is equal to zero.

Example 6: Find the slope of the line represented by the equation $x = 2$.

We will again start by graphing this line:



This time, let's begin by discussing how to use the formula slope = $\frac{y_2 - y_1}{x_2 - x_1}$. We will pick the point $(2, 4)$ to be our (x_1, y_1) , and the point $(2, 1)$ will be our (x_2, y_2) .

When we plug these numbers into the formula, we find that slope = $\frac{1-4}{2-2} = \frac{-3}{0}$.

However, we have a problem because there is no such thing as -3 divided by zero. Therefore, we say that the slope is *undefined* or that the line has *no slope*. (By the way, if you get confused about whether the undefined one has a zero on the top or the bottom, then try plugging your fractions into a calculator. If you remember that a fraction bar can also mean "divided by," then you will find that you will get an error when you try to plug in $\frac{-3}{0}$, whereas plugging in $\frac{0}{-3}$ will give you zero, not an error.)

Of course, you can still use the formula slope = $\frac{\text{rise}}{\text{run}}$. If you pick two different points, your rise will be a number other than zero, and your run will be zero. As you should have already guessed, this tells us that the slope of the line $x = 2$ is undefined.

Here are some hints to help you check your work:

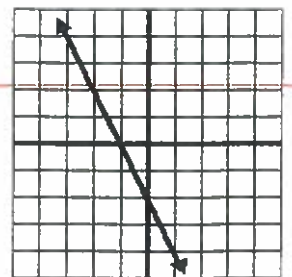
- (a) The slope of a line that goes up from left to right (like the one in Example 3) will always be positive.
- (b) The slope of a line that goes down from left to right (like the one in Example 4) will always be negative.
- (c) The slope of a horizontal line (like the one in Example 5) will always be zero.
- (d) The slope of a vertical line (like the one in Example 6) will always be undefined.

Problems:

39. Find the intercepts of the line at the right.

x-intercept(s) -1

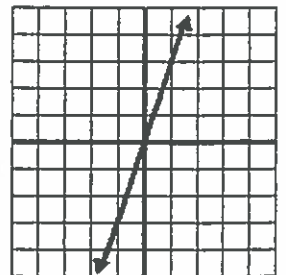
y-intercept(s) -2



40. Find the intercepts of the line at the right.

x-intercept(s) _____

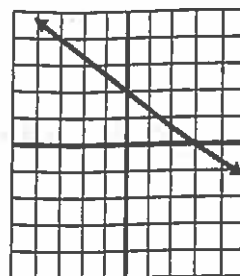
y-intercept(s) _____



41. Find the intercepts of the line at the right.

x-intercept(s) _____

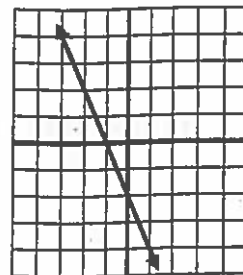
y-intercept(s) _____



42. Find the intercepts of the line at the right.

x-intercept(s) _____

y-intercept(s) _____



43. Find the intercepts of the curve at the right.

x-intercept(s) _____

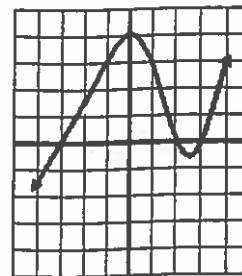
y-intercept(s) _____



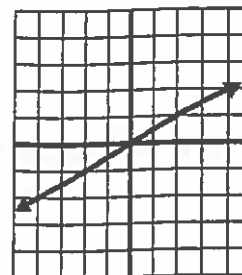
44. Find the intercepts of the curve at the right.

x-intercept(s) _____

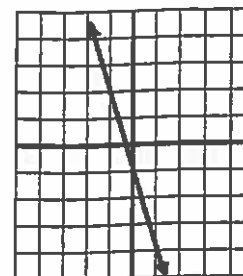
y-intercept(s) _____



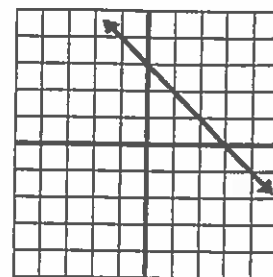
45. Find the slope of the line at the right. _____



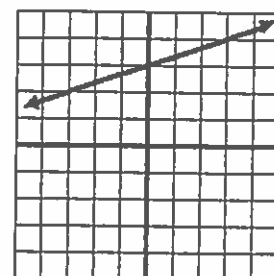
46. Find the slope of the line at the right. _____



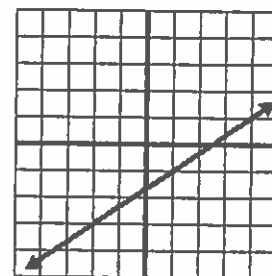
47. Find the slope of the line at the right. _____



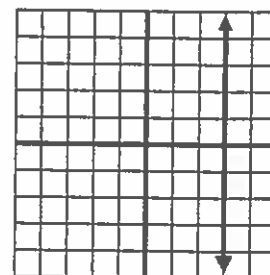
48. Find the slope of the line at the right. _____



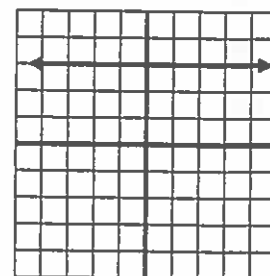
49. Find the slope of the line at the right. _____



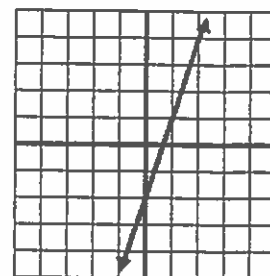
50. Find the slope of the line at the right. _____



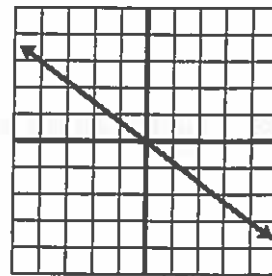
51. Find the slope of the line at the right. _____



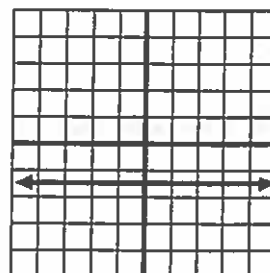
52. Find the slope of the line at the right. _____



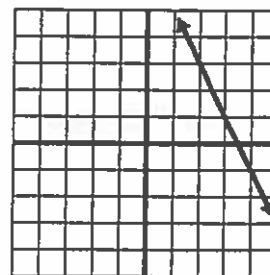
53. Find the slope of the line at the right. _____



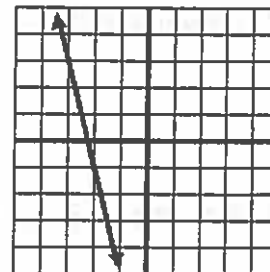
54. Find the slope of the line at the right. _____



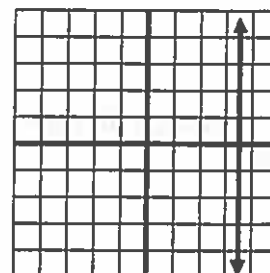
55. Find the slope of the line at the right. _____



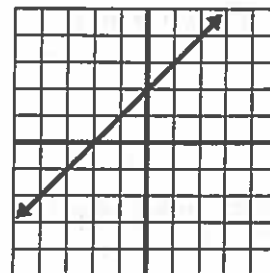
56. Find the slope of the line at the right. _____



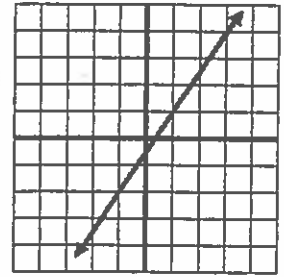
57. Find the slope of the line at the right. _____



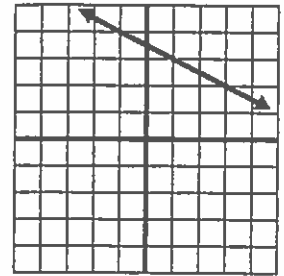
58. Find the slope of the line at the right. _____



59. Find the slope of the line at the right. _____



60. Find the slope of the line at the right. _____



61. Find the slope of the line that passes through the points $(1, 0)$ and $(4, -1)$. _____

62. Find the slope of the line that passes through the points $(-5, 8)$ and $(3, -2)$. _____

63. Find the slope of the line that passes through the points $(-3, 2)$ and $(2, -3)$. _____

64. Find the slope of the line that passes through the points $(-3, 4)$ and $(-1, -6)$. _____

65. Find the slope of the line that passes through the points $(2, 0)$ and $(8, -5)$. _____

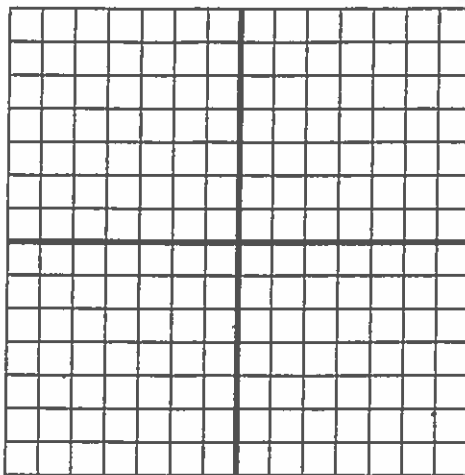
66. Find the slope of the line that passes through the points $(4, -3)$ and $(-3, 1)$. _____

67. Find the slope of the line that passes through the points (5, 2) and (1, -4). _____
68. Find the slope of the line that passes through the points (7, -6) and (-2, -5). _____
69. Find the slope of the line that passes through the points (-3, 1) and (-1, -2). _____
70. Find the slope of the line that passes through the points (-5, 2) and (3, 2). _____
71. Find the slope of the line that passes through the points (-3, 0) and (2, 2). _____
72. Find the slope of the line that passes through the points (-3, 5) and (-3, 4). _____
73. Find the slope of the line that passes through the points (2, 1) and (4, 1). _____
74. Find the slope of the line that passes through the points (4, 3) and (-2, 0). _____
75. Find the slope of the line that passes through the points (1, 0) and (1, -1). _____

76. Find the slope and y-intercept of the line described by the equation $2x + y = 4$ by graphing the line.

slope _____

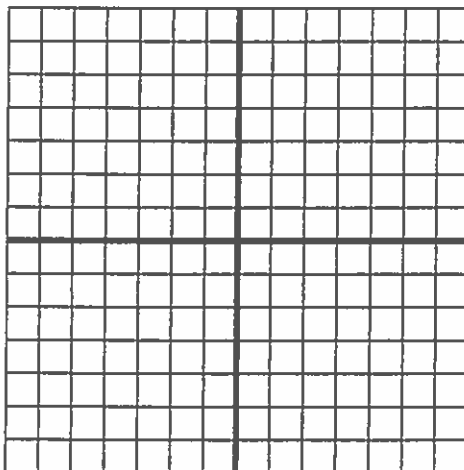
y-intercept _____



77. Find the slope and y-intercept of the line described by the equation $0.5x + y = 2$ by graphing the line.

slope _____

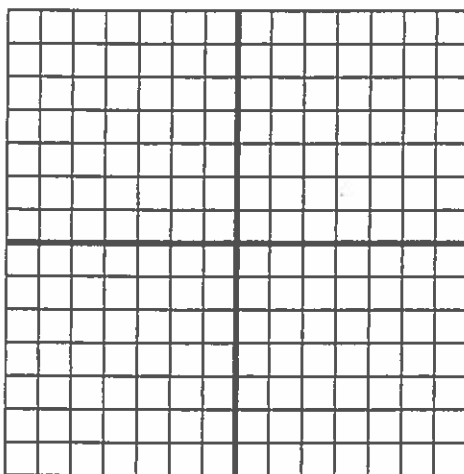
y-intercept _____



78. Find the slope and y-intercept of the line described by the equation $0.2x - y = 3$ by graphing the line.

slope _____

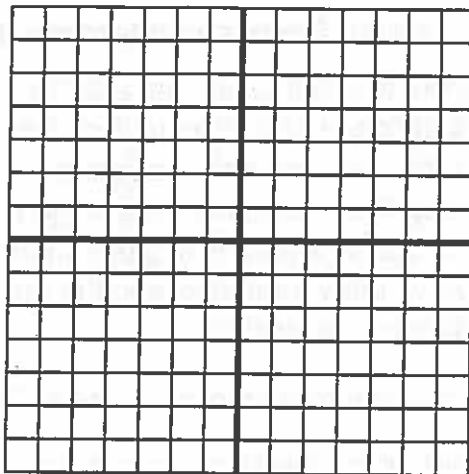
y-intercept _____



79. Find the slope and y-intercept of the line described by the equation $5x - 2y = 0$ by graphing the line.

slope _____

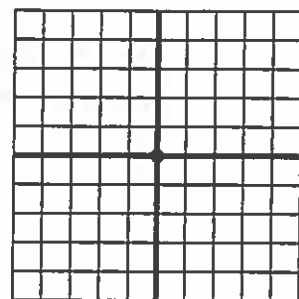
y-intercept _____



Example 2: Graph the line represented by the equation $y = 3x$.

Step 1. Plot the y-intercept.

In this case, we are not actually told what the y-intercept is. However, note that saying $y = 3x + 0$ is exactly the same thing as saying $y = 3x$, and so we know our y-intercept is 0.

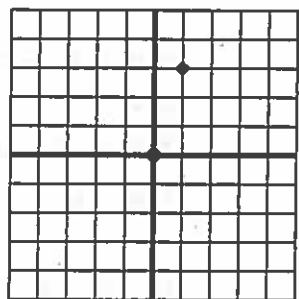


Step 2. Find the slope. From the y-intercept, rise the proper amount, and then run the proper amount.

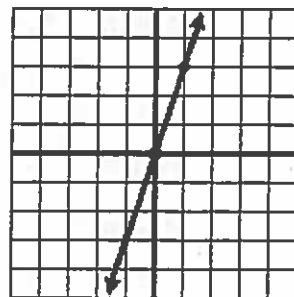
Note that, for this example, the slope is equal to 3.

Also note that we can say that $3 = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$. So,

from the y-intercept that we just plotted, we will rise 3 and run 1.



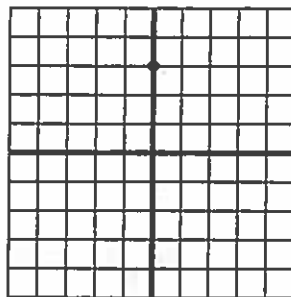
Step 3. Draw a straight line connecting your points, and put arrows on the ends to show that the line never stops.



Example 3: Graph the line represented by the equation $y = -x + 3$.

Step 1. Plot the y-intercept.

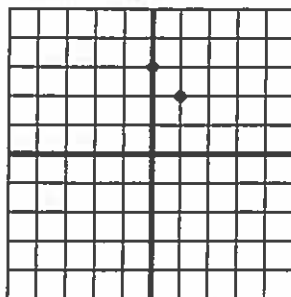
In this case, the y-intercept is 3.



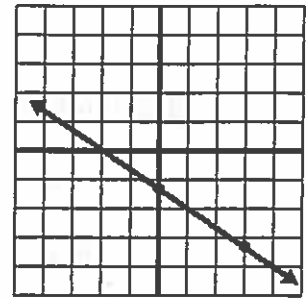
Step 2. Find the slope. From the y-intercept, rise the proper amount, and then run the proper amount.

For this example, the slope is equal to -1 (recall that $-x = -1x$). Now, we need to note that $-1 = \frac{-1}{1} = \frac{\text{rise}}{\text{run}}$.

So, from the y-intercept that we just plotted, we rise -1 (which means that we will go *down* 1) and run 1.



Step 3. Draw a straight line connecting your points, and put arrows on the ends to show that the line never stops.



Problems:

80. Find the slope and the y-intercept of the line described by the equation $y = 2x + 1$.

slope _____

y-intercept _____

81. Find the slope and the y-intercept of the line described by the equation $y = -x - 5$.

slope _____

y-intercept _____

82. Find the slope and the y-intercept of the line described by the equation $y = 2x - 3$.

slope _____

y-intercept _____

83. Find the slope and the y-intercept of the line described by the equation $y = \frac{3x - 2}{5}$.

slope _____

y-intercept _____

84. Find the slope and the y-intercept of the line described by the equation $y = -\frac{5}{6}x + 4$.

slope _____

y-intercept _____

85. Find the slope and the y-intercept of the line described by the equation $y = 3 - 2x$.

slope _____

y-intercept _____

86. Find the slope and the y-intercept of the line described by the equation $3x + 2y = 5$.

slope _____

y-intercept _____

87. Find the slope and the y-intercept of the line described by the equation $2x - 4y = 3$.

slope _____

y-intercept _____

88. Find the slope and the y-intercept of the line described by the equation $2x - y = 0$.

slope _____

y-intercept _____

89. Find the slope and the y-intercept of the line described by the equation $x - 3y + 3 = 0$.

slope _____

y-intercept _____

90. Find the slope and the y-intercept of the line described by the equation $2x + 3y = -2$.

slope _____

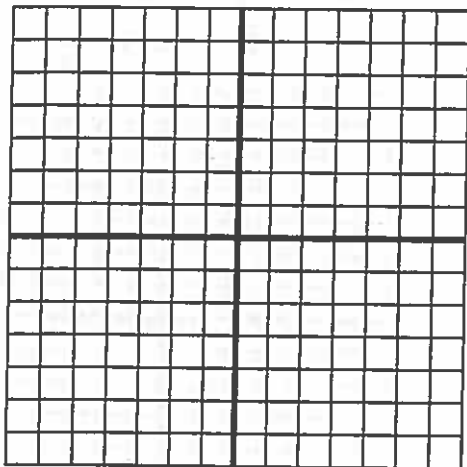
y-intercept _____

91. Find the slope and the y-intercept of the line described by the equation $10x - y = -5$.

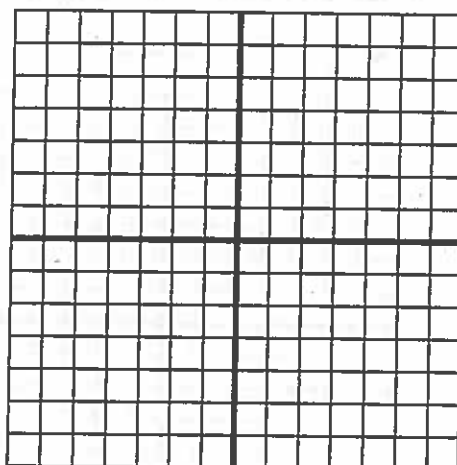
slope _____

y-intercept _____

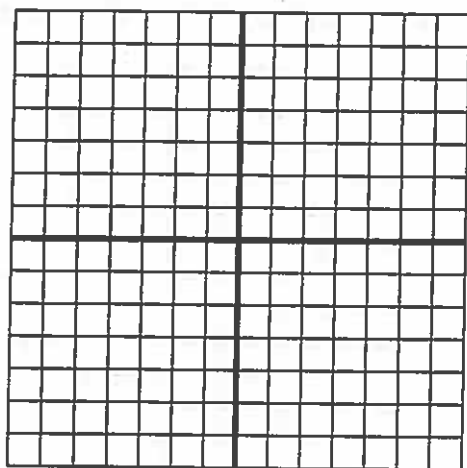
92. Graph the line with a y-intercept of -2 and a slope of $\frac{2}{3}$.



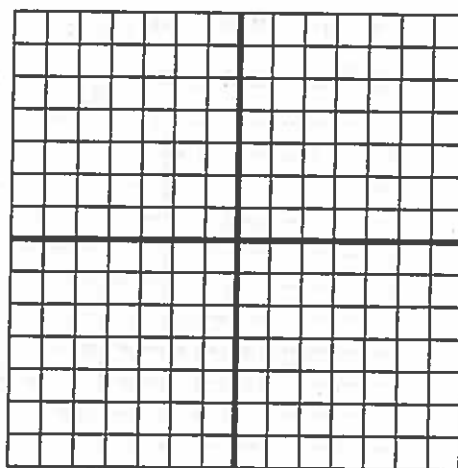
95. Graph the line with a y-intercept of -2 and a slope of $-\frac{1}{2}$.



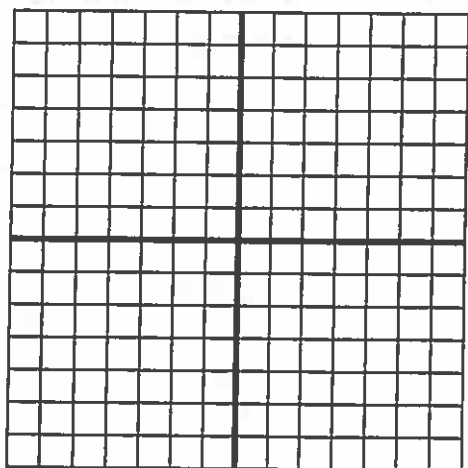
93. Graph the line with a y-intercept of -1 and a slope of -2.



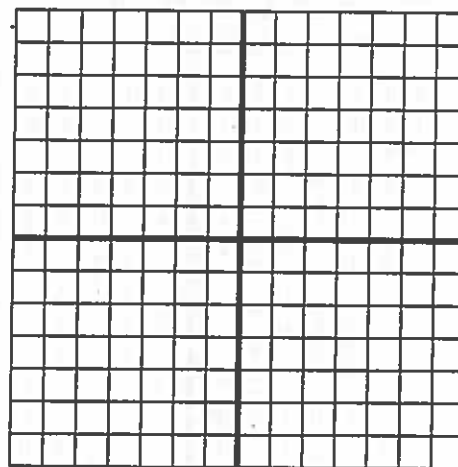
96. Graph the line with a y-intercept of -3 and a slope of 2.



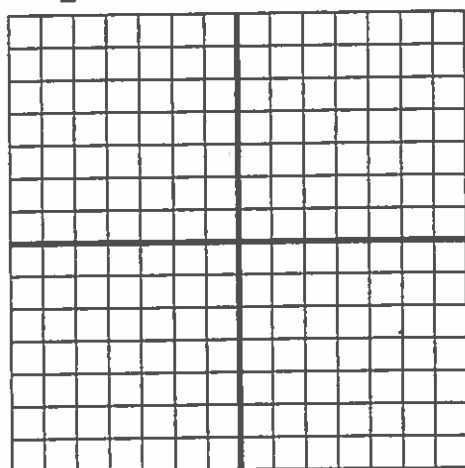
94. Graph the line with a y-intercept of 1 and a slope of $-\frac{4}{3}$.



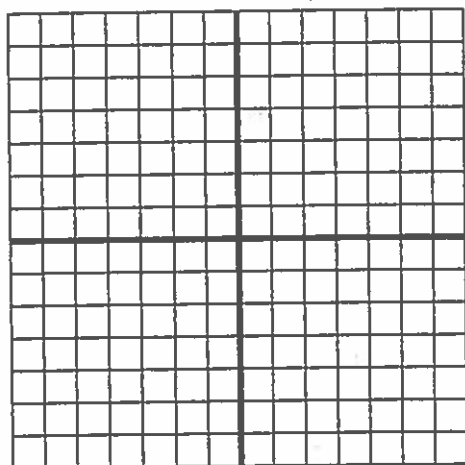
97. Graph the line with a y-intercept of $-\frac{4}{5}$ and a slope of -1.



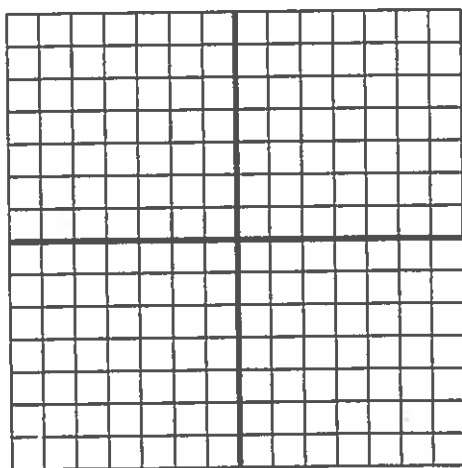
98. Graph the line with a y-intercept of $\frac{1}{2}$ and a slope of 3.



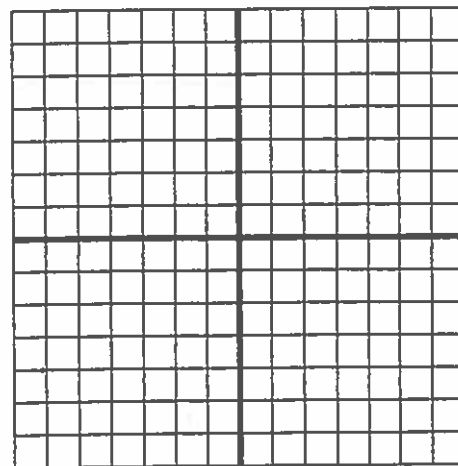
99. Graph the line with a y-intercept of 2 and a slope of $\frac{3}{4}$.



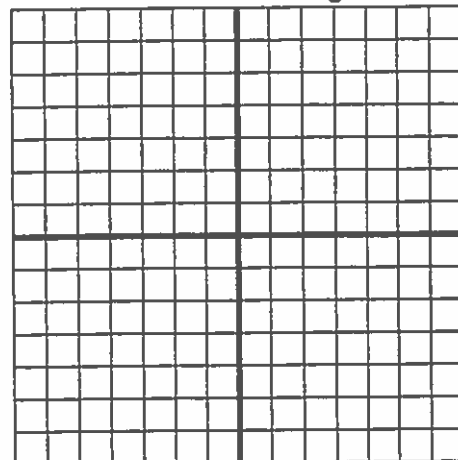
100. Graph by finding the slope and the y-intercept: $y = 3x - 2$



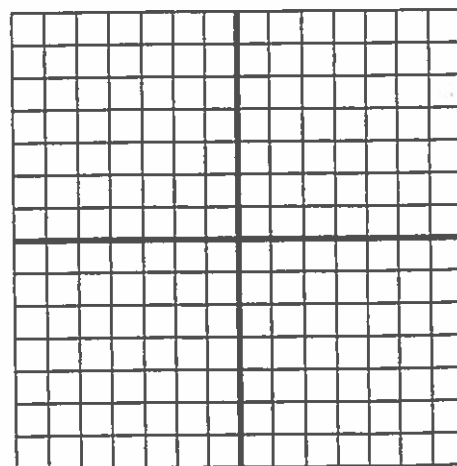
101. Graph by finding the slope and the y-intercept: $y = \frac{3}{4}x - 2$



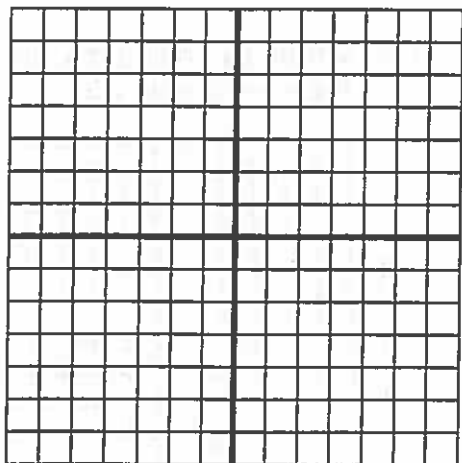
102. Graph by finding the slope and the y-intercept: $y = \frac{2}{3}x + 1$



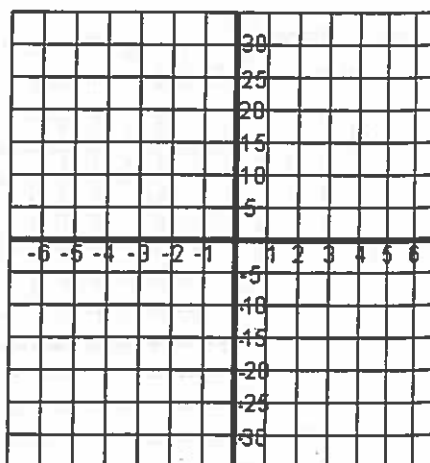
103. Graph by finding the slope and the y-intercept: $y = -3x - 1$



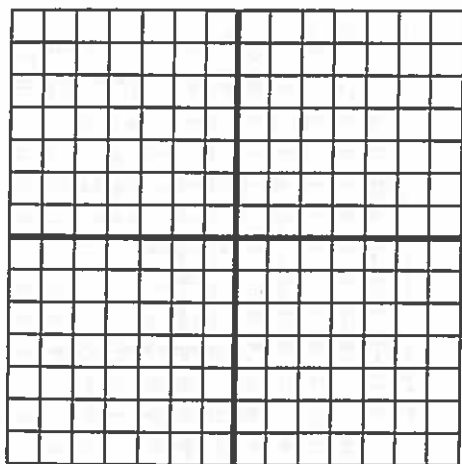
104. Graph by finding the slope and the y-intercept: $y = x - 5$



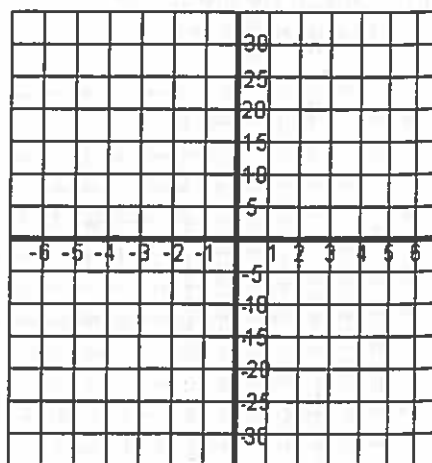
107. Graph by finding the slope and the y-intercept: $y = 2x + 10$



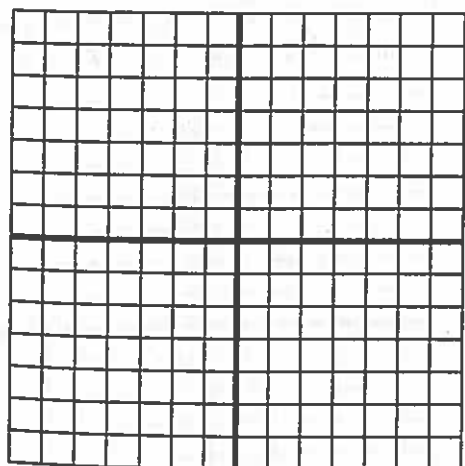
105. Graph by finding the slope and the y-intercept: $y = \frac{-1}{2}x + 2$



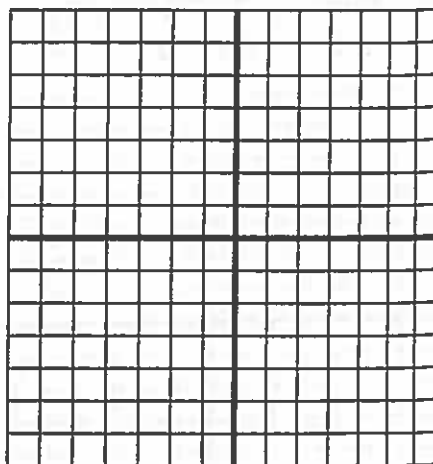
108. Graph by finding the slope and the y-intercept: $y = -5x - 15$



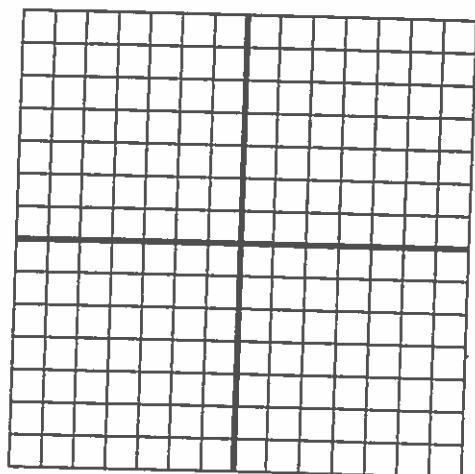
106. Graph by finding the slope and the y-intercept: $y = -\frac{2}{5}x - 3$



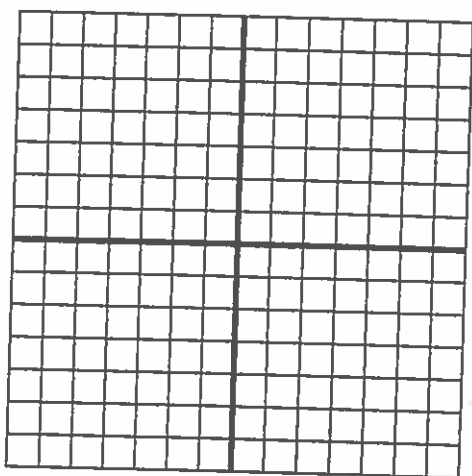
109. Graph by finding the slope and the y-intercept: $y = 5 - 2x$



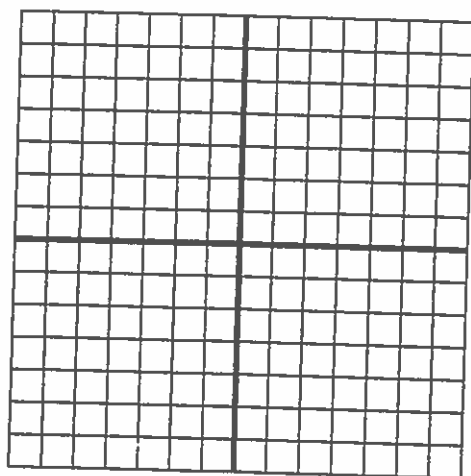
110. Graph by finding the slope and the y-intercept: $y = -3x + 0.5$



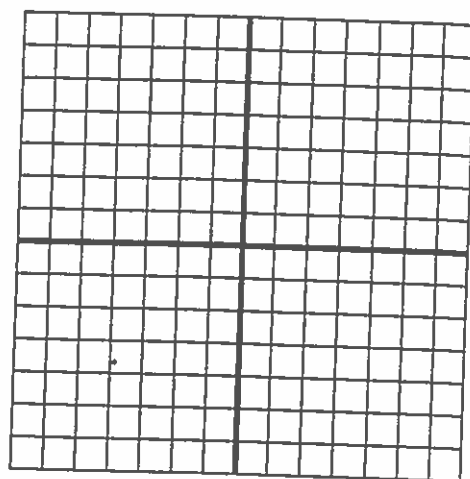
111. Graph by finding the slope and the y-intercept: $y = -x + 5$



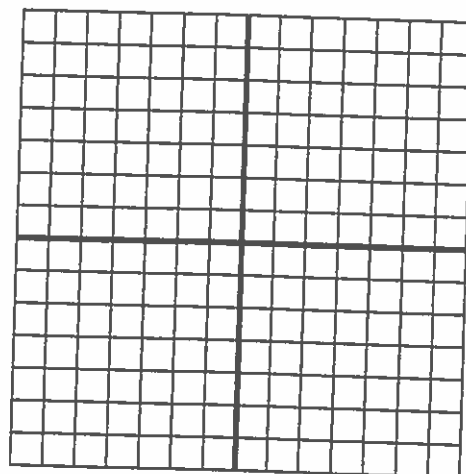
112. Graph by finding the slope and the y-intercept: $y = 1 - 3x$



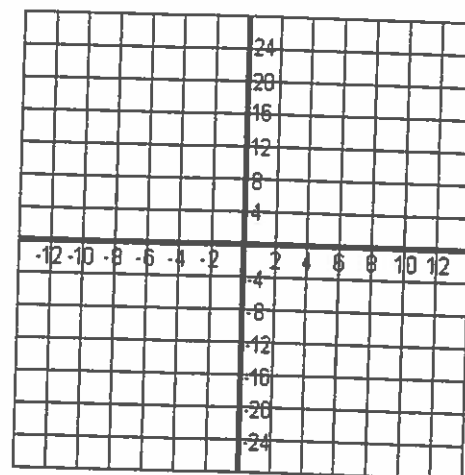
113. Graph by finding the slope and the y-intercept: $y = -2x$



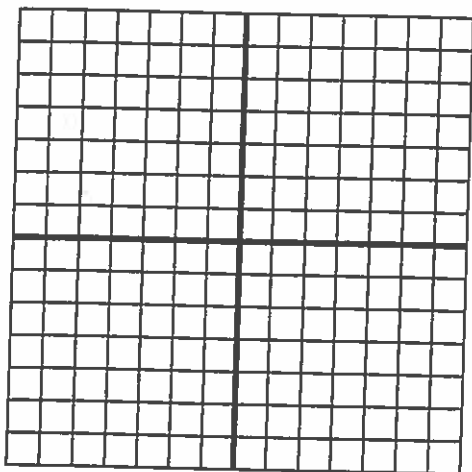
114. Graph by finding the slope and the y-intercept: $3x + 2y = 1$



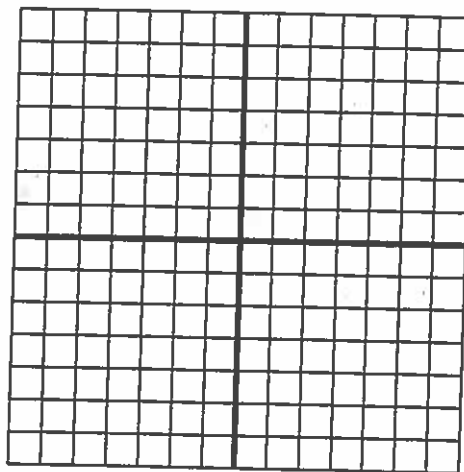
115. Graph by finding the slope and the y-intercept: $3x - 2y = 5$



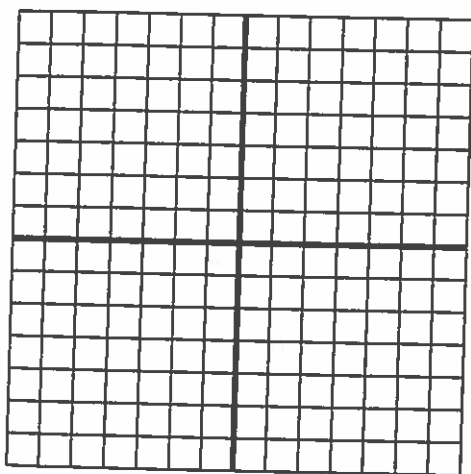
116. Graph by finding the slope and the y-intercept: $x + 2y = 0$



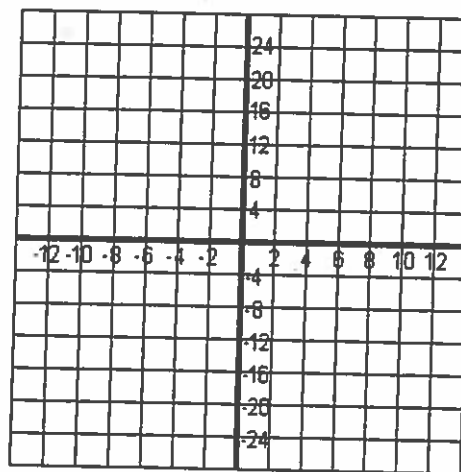
119. Graph by finding the slope and the y-intercept: $x + 2y + 3 = 0$



117. Graph by finding the slope and the y-intercept: $3x + 4y = -1$



120. Graph by finding the slope and the y-intercept: $4x + 5y + 3 = 0$



118. Graph by finding the slope and the y-intercept: $3x - y + 2 = 0$

